Seasonal Variability of Rainfall Extremes

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ABSTRACT

Monthly rainfall extremes have been analyzed for three stations in Southern Ontario. The double exponential probability distribution was fitted to the extreme values for each month considered, each duration selected, and sets of annual extremes. A station-year approach yielded monthly and annual extreme value distributions for the lumped region of Southern Ontario. The analysis has revealed a pronounced seasonal pattern in the rainfall extremes – the amount of rain expected with a selected probability of occurrence during the summer being considerably greater than the rainfall that might be expected to be exceeded at the same probability level during the spring or fall. The extent of the seasonal variability was found also to vary with duration. The implications of the variability are seen to be significant for the estimation of the magnitude and frequency of floods.

1 Introduction

Data regarding extreme rainfalls are often very useful for hydrologists and water resources engineers. Rainfall magnitude and frequency information is used extensively for the prediction of design floods and the probability of occurrence of such flows.

Historically, flood prediction approaches have equated flood frequency to that of the associated rainfall event (Chow, 1962; Spencer and Dickinson, 1970). Researchers have demonstrated that, at least in rural areas, these approaches prove to be problematic as there is virtually no correlation between rainfall and runoff frequencies for selected events (Hiemstra and Reich, 1967; Matheson, 1975). In fact, as the manner in which the watershed system operates upon rainfall input varies dramatically from season to season, relationships between rainfall and runoff frequencies must be dependent upon seasonal variables.

Therefore, to identify aspects of rainfall input relevant to watershed response, it is important to consider the seasonal variability of extreme rainfall characteristics. A method of approaching this task is presented below with reference to data for Southern Ontario.

2 Monthly rainfall extremes

Monthly rainfall extremes of 5-, 10-, 15-, 30-, 60-, 120-, 360-, 720-, and 1440-min durations have been analyzed for three stations in Southern Ontario. The

TABLE 1. Correlation coefficients for Gumbel extremal distributions fitted to monthly and annual rainfall extremes in Southern Ontario

Duration: minutes	Time Interval								
	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Year
10	0.975	0.946	0.976	0.990	0.960	0.951	0.977	0.980	0.977
30	0.983	0.949	0.978	0.983	0.952	0.921	0.970	0.984	0.968
60	0.986	0.948	0.966	0.992	0.965	0.912	0.958	0.970	0.978
360	0.978	0.969	0.966	0.973	0.983	0.963	0.960	0.979	0.977
720	9.978	0.978	0.962	0.982	0.988	0.972	0.969	0.980	0.986
1440	0.975	0.987	0.955	0.986	0.979	0.970	9.965	0.982	0.984

monthly extremal information has been available for the months of April through November for London, St Thomas, and Toronto for periods of 35, 50, and 35 years respectively. Data for the winter months of December through March were not analyzed as most precipitation during this season is in the form of snow. Extreme rainfalls in these months are rare and are generally small in comparison to amounts occurring during the rest of the year.

The Gumbel (double exponential) distribution was fitted to the extreme values for each month considered and each duration selected. The sets of annual extremes for the durations at each station were also fitted with the same form of distribution. Fitting was achieved in all cases by the method of moments, and the goodness of fit was examined with regard to both correlation coefficients and control curves. The average correlation coefficients shown in Table 1 reveal that the Gumbel distribution represents yearly and monthly rainfall extremes equally well.

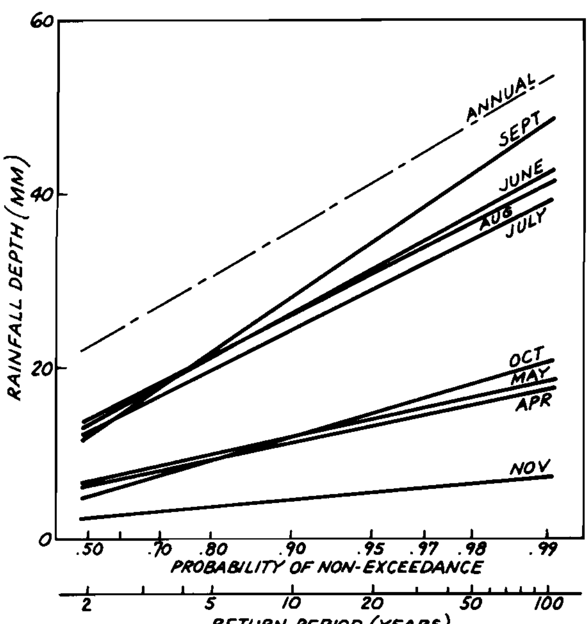

Fig. 1 Monthly and annual extreme value distributions for 30-min storm rainfall at London, Ontario.

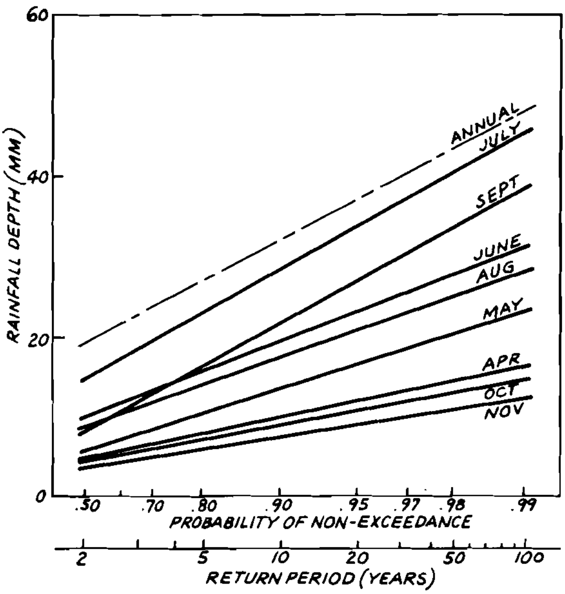

Fig. 2 Monthly and annual extreme value distributions for 30-min storm rainfall at Toronto, Ontario.

Although there has been no attempt in the study to fit other distributions to the rainfall extremes, consideration was given to the σ_1/σ_2 ratio referred to by Jenkinson (1955) and Krishnan and Kushwaha (1975), where σ_1 is the standard deviation of the set of extreme values and σ_2 is the standard deviation of the greater members in pairs of extremes. The Gumbel distribution is most applicable when $\sigma_1 = \sigma_2$, tending to yield overestimates when $\sigma_1 > \sigma_2$ and underestimates when $\sigma_1 < \sigma_2$. Determinations of the ratio for data sets for all months and the range of durations revealed an average ratio of 1.00 and no apparent trends with either season or duration. Sampling errors and the presence of outlier points in the data appeared to be the causes of variations in the ratio. From these results, it follows that the Gumbel distribution should represent the data well.

Examples of the fitted monthly and annual extreme value distributions are presented in Figs 1 and 2. Although the periods of record considered were 35 to 50 years in length, the results revealed considerable scatter attributable to sampling error. In order to reduce such effects, the data for the three locations were combined by means of a station-year approach to yield monthly and annual extreme value distributions for the lumped region of Southern Ontario. An example of these results is presented in Fig. 3.

3 Seasonal variability of parameters

Figs 1, 2, and 3 reveal that for the climatic region studied there is a pronounced seasonal pattern of rain extremes. The return period associated with selected rainfall amounts occurring during the summer months is considerably smaller than that associated with the same amounts occurring during the spring or fall. For example, the return period for a 20 mm rainfall occurring in 30

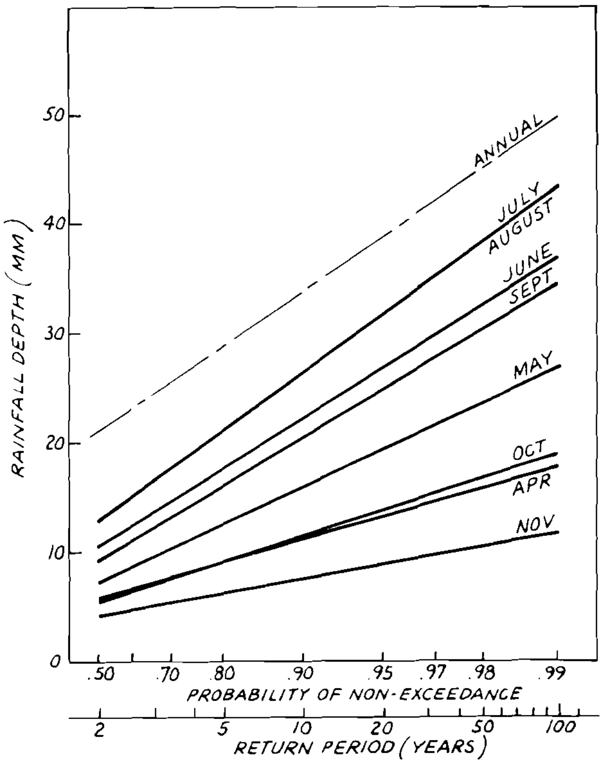

Fig. 3 Monthly and annual extreme value distributions for 30-min storm rainfall in Southern Ontario.

minutes at a point in Toronto during a particular summer month is approximately 10 years; for the same event occurring in a spring or fall month, the return period is >100 years. The amount of rain expected with a selected probability of occurrence during the summer is about double the rainfall that might be expected to be exceeded at the same probability level during the spring and fall.

The seasonal pattern was found to be the same for all durations, with the extent of variability a function of duration. The effect of duration is revealed in the variability of the means and standard deviations of the monthly extremal data, presented in Figs 4 and 5 respectively. The extremal distribution for any selected month and duration may be determined from the relationship,

$$P[X \leqslant x] = e^{-e^{-(a+x)/c}}$$

where $P[X \leqslant x]$ = the probability of the variate X being equal to or smaller than the value x,

$a = 0.450\sigma_x - \mu$,

$c = 0.780\sigma_x$,

μ = the mean of the monthly extreme rainfall, and

σ_x = the standard deviation of the monthly extreme rainfall.

Although the range of seasonal variability among the means appears most peaked for the middle durations in Fig. 4, the ratios of the means in relation to

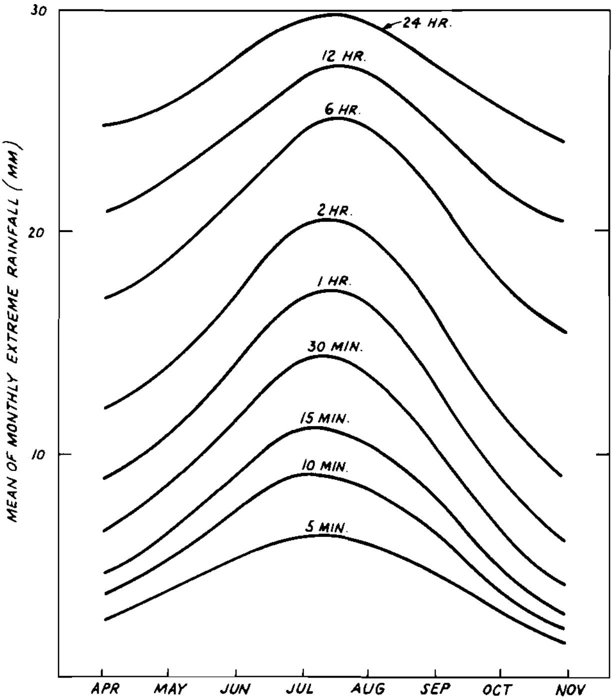

Fig. 4 Means of monthly extreme rainfall distributions for selected storm durations in Southern Ontario.

the April means reveals in Fig. 6 that the range decreases continuously as duration increases. The mean summer extreme rainfall of short duration is more than twice the April mean extreme, while the summer 24 hourly mean extreme is very similar to the April mean.

Fig. 7 reveals that the range in standard deviations of monthly extreme rainfalls is greatest for the 30-, 60-, and 120-min durations. Although the slope of the monthly extremal distributions increases continuously with duration, the range of slopes decreases as duration grows shorter and longer than one hour.

The seasonal pattern in rainfall extremes is attributable to an increase in short-duration convective storm activity during the months of June through September. The pattern in Fig. 7 would also suggest that the storm durations for which this activity is most pronounced is between 30 min and 2 hours. There is also evidence that, for rainfalls occurring in durations of less than 6 hours and during the months of April through November, rainfall extremes are least in November.

The seasonal variability of extremes in conjunction with storm duration can be summarized in the form of monthly depth vs duration curves for selected return periods. An example of this format is shown in Fig. 8.

4 Annual extremes from monthly data

If a year is considered to be composed of monthly intervals, and if for each interval there exists an independent extremal probability distribution for rain-

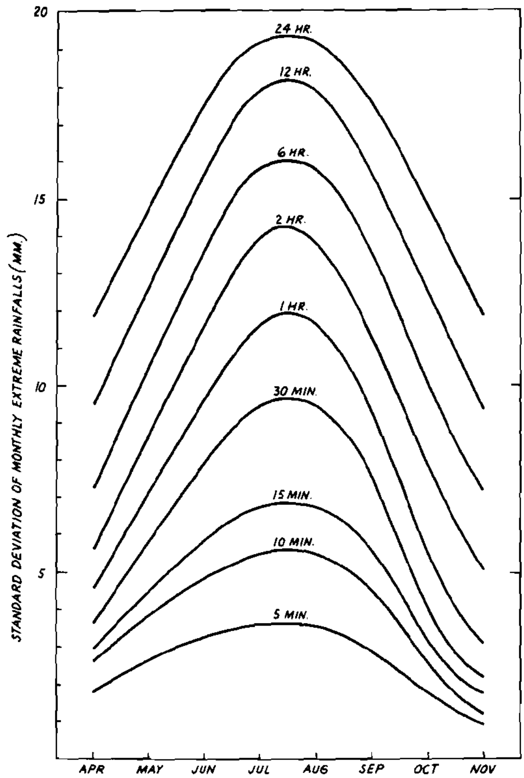

Fig. 5 Standard deviations of monthly extreme rainfall distributions for selected storm durations in Southern Ontario.

fall amounts occurring in specific durations, the probability of a selected rainfall event not being exceeded in a year may be expressed as the product of the monthly non-exceedance probabilities. That is,

$$P_y[X < x] = P_1[X < x] \times P_2[X < x] \times \ldots \times P_n[X < x]$$
$$= \prod_{i=1}^{n} P_i[X < x] \tag{1}$$

where $P_y[X < x]$ = the probability of x not being exceeded in a year, i.e. n time intervals, and

$P_i[X < x]$ = the probability of x not being exceeded in the ith time interval.

Then the probability of x being equalled or exceeded in a year may be expressed,

$$P_y[X \geqslant x] = 1 - \prod_{i=1}^{n} P_i[X < x]\,. \tag{2}$$

When monthly rainfall extremes are represented with the Gumbel distribution, the probability of a selected rainfall event being equalled or exceeded in a year may then be expressed,

$$P_y[X \geqslant x] = 1 - \prod_{i=1}^{n} e^{-e^{-(x+a_i)/c_i}} \tag{3}$$

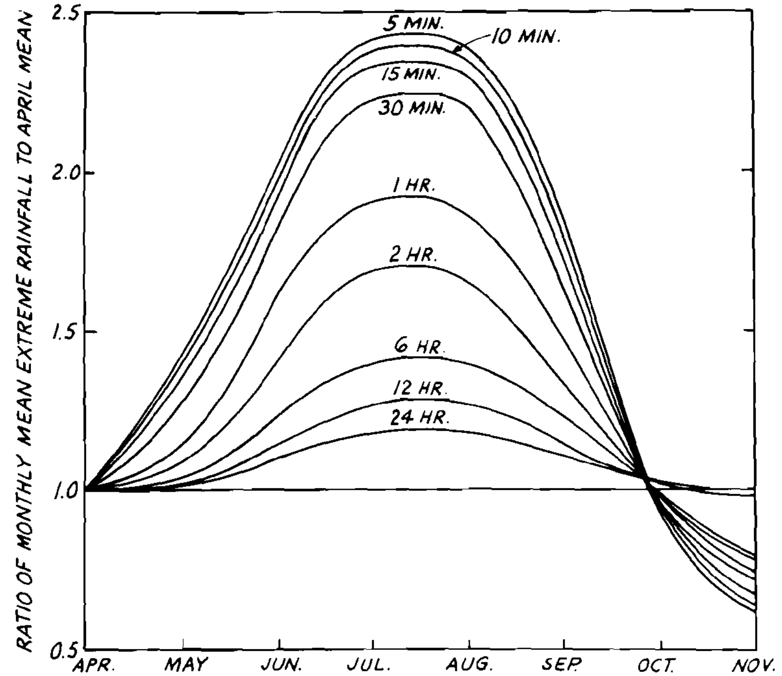

Fig. 6 Ratios of monthly mean extremes for selected storm durations in Southern Ontario.

where a_i, c_i = the parameters of the Gumbel double exponential distribution representing the extremes in the ith time interval.

This approach was employed to estimate the distribution of annual rainfall extremes for each duration considered at the Southern Ontario stations. Examples of the results are shown in Figs 1, 2, 3, and 8.

The annual extremal distributions developed with expression (3) are not of double exponential form. However, except for data sets in which serious outliers exist, these distributions closely approximate double exponential distributions fitted to annual extremes. Where outliers do exist, the approach involving expression (3) greatly reduces the effect of outliers on the estimated annual distribution.

5 Summary comments

Short-duration convective storm activity during the summer months causes a pronounced seasonal pattern in extreme rainfall amounts in Southern Ontario. The amount of rain expected with a selected probability of occurrence during the summer is considerably greater than the rainfall that might be expected to be exceeded at the same probability level during the spring or fall. Although the pattern is similar for all storm durations from 5 minutes to 24 hours, the extent of the seasonal variability varies with duration. Variations in slope of the monthly extreme rainfall relationships are greatest for a storm duration of one hour.

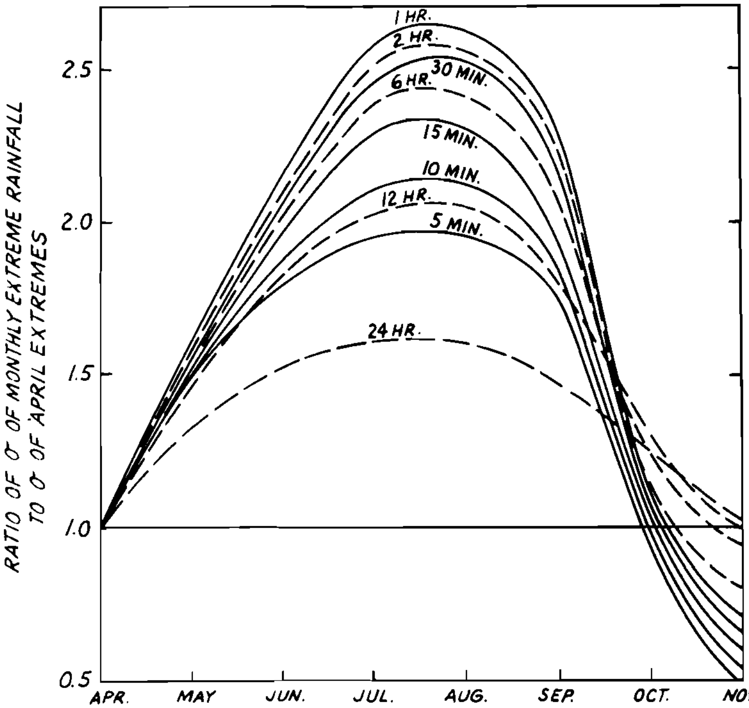

Fig. 7 Ratios of monthly standard deviations of extremes for selected storm duration in Southern Ontario.

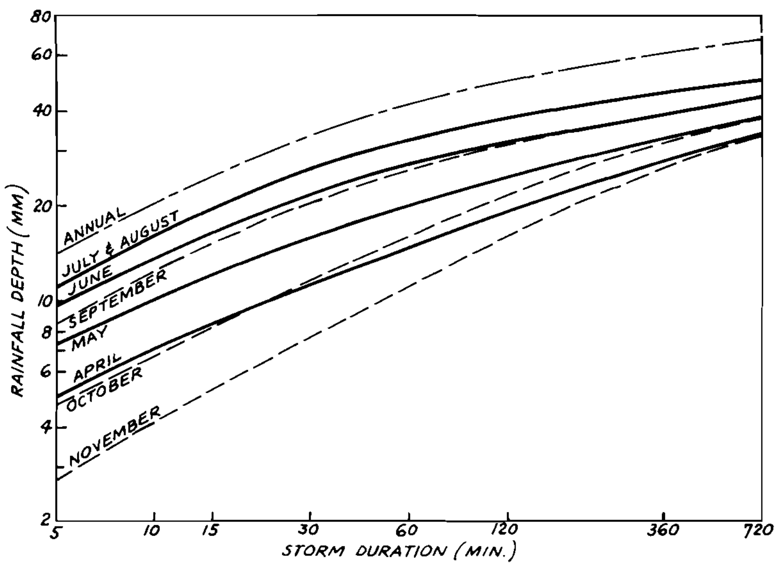

Fig. 8 Monthly depth vs duration curves for storms of 10-year return period in Southern Ontario.

The implications of this seasonal variability are significant for the estimation of flood peaks and their frequency of occurrence. Although rainfall extremes are most likely to occur during the summer months, peak streamflow in Southern Ontario occurs most frequently during the winter and spring period. Therefore, watershed moisture and cover conditions significantly moderate summer rainfalls but exert a much smaller effect on spring storms. Information regarding the nature of rainfall extremes in the spring months may lead to improved understanding and prediction of the runoff vs rainfall relationships not only during that period but also throughout the year.

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