A Revised Method for Determining the Direct and Diffuse Components of the Total Short-wave Radiation

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ABSTRACT

A relationship, derived by Liu and Jordan (1960), under which the total short-wave radiation may readily be subdivided into its direct and diffuse components is shown to vary both spatially and seasonally. This variability is attributed to changes in the importance of the multiple reflection of short-wave radiation between the earth's surface and atmosphere. A revised relationship, which incorporates the influence of this process, is shown to have applicability at a large number of Canadian locations.

1 Introduction

Increased utilization of solar energy in the 1970s has resulted in greater interest in the amount of short-wave radiation (wavelengths $<4.0\ \mu m$) incident upon a surface of given orientation, as specified by its tilt and aspect. However, the more readily available observational data are characteristically for the more restricted, and commonly less appropriate, horizontal surface. Kondratyev (1969) and others have presented methods for calculating the radiation on a non-horizontal surface given the energy incident on a horizontal surface, but the approach requires a knowledge of the proportions of the total short-wave radiation ($K\downarrow$) reaching the surface as direct ($S\downarrow$) and diffuse ($D\downarrow$) short-wave radiation. These components are routinely measured at only a few locations in comparison to the more widespread measurements of the total short-wave radiation, but analyses of some of the available data have shown that a relationship between the short-wave radiation and its component fluxes does exist (e.g. Liu and Jordan, 1960). More complex calculation methods have been evaluated (e.g. Sadler, 1975), but the simplicity of the Liu and Jordan approach has led to its widespread adoption (e.g. Hunn et al., 1975; Löf and Tybout, 1973).

However, the Liu and Jordan model was derived from data for only one location (Blue Hill, Massachusetts) and has had little independent testing. Ruth and Chant (1976) tested the relationship using available data for four Canadian locations and found that the Liu and Jordan approach generally underestimated the ratio of $D\downarrow$ to $K\downarrow$ for a given ratio of $K\downarrow$ to K_0 (the extra-terrestrial short-wave radiation). They attributed the discrepancy to a latitude dependence in the relationship, though the physical factors involved were not

TABLE 1. Canadian stations with regular measurements of diffuse and total short-wave radiation

	Latitude (N)		Longitude (W)		Record Used
	°	′	°	′	
Toronto Met. Res. Stn	43	48	79	33	1967–1975
Toronto Scarborough	43	43	79	14	1960–1967
Montreal Jean Brébeuf	45	30	73	37	1965–1974
Goose Bay	53	18	60	27	1962–1975
Resolute	74	43	94	59	1960–1974

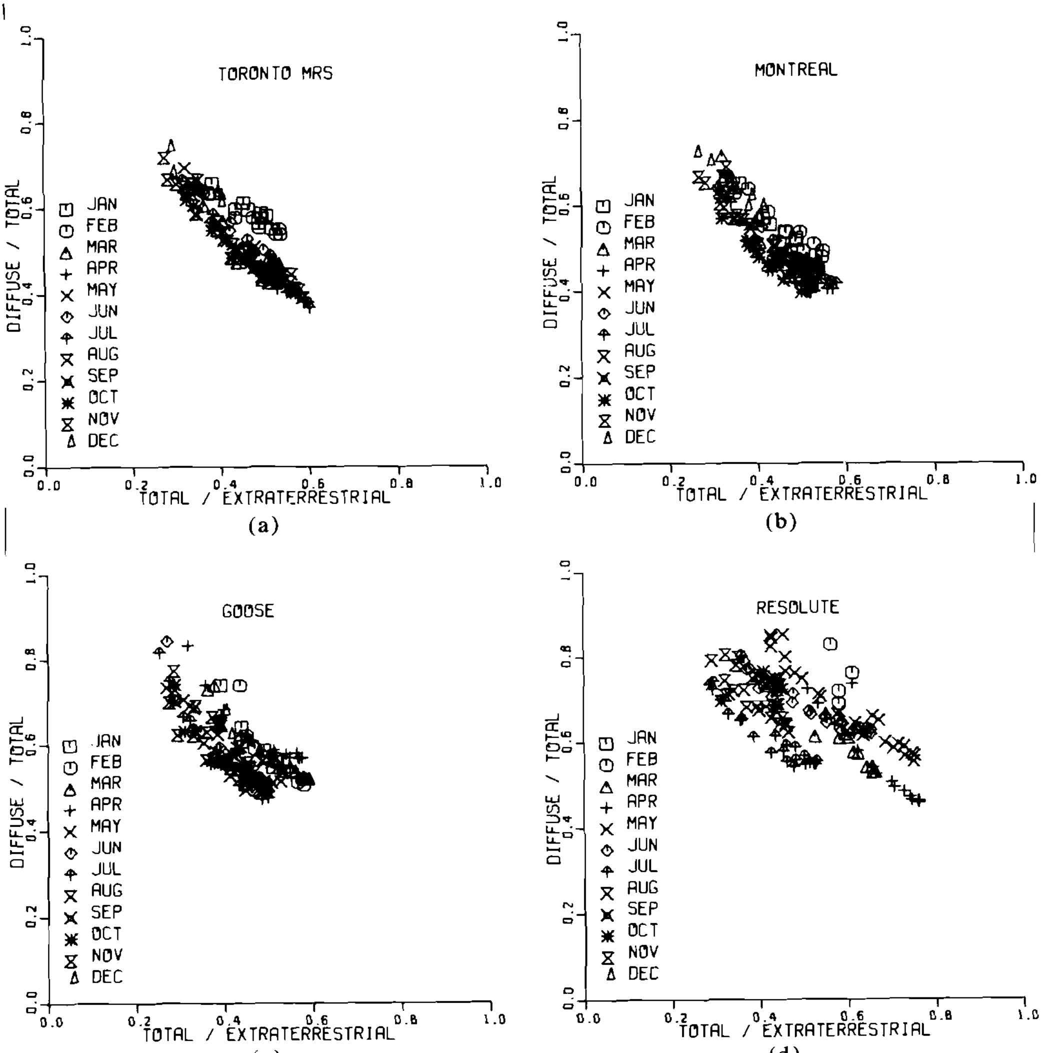
discussed. It is the purpose of this paper to evaluate the performance of the Liu and Jordan model at selected Canadian locations, to provide reasons for its demonstrated inapplicability, and to describe and substantiate the validity of a revised method for determining the component fluxes of the total short-wave radiation.

Observational data from five locations in Canada are available to investigate the relationship between the total short-wave radiation and its direct and diffuse components. The data are collected and published by the Canadian Atmospheric Environment Service. Table 1 provides details of the data record used in the present study. A departure from the computational procedures used by Liu and Jordan is that this study uses mean hourly rather than mean daily values for each month. The benefit of presenting averages for a shorter time interval is that the values of $S{\downarrow}$ and $D{\downarrow}$ determined from the relationship are commonly used as input to equations with a validity for hourly, or even shorter, time periods. The alternative is to determine daily values of $S{\downarrow}$ and $D{\downarrow}$ and then to estimate hourly values from the daily totals (Duffie and Beckman, 1974). The current wider availability of hourly values increases the desirability of working directly at the more appropriate time scale (rather than deriving them by a much more devious method).

2 Evaluation of the Liu and Jordan approach

Fig. 1 presents the data for four Canadian locations in a format similar to that used by Liu and Jordan in their analysis of the Blue Hill data. Due to the unreliability at low radiative fluxes of both the measured values and, to a greater extent, the ratios they determine, data have not been plotted when $K{\downarrow}$ was less than 42 $\mathrm{kJm^{-2}h^{-1}}$. The value for K_0 was based on a solar constant of 1354 W $\mathrm{m^{-2}}$ and calculated as a mean hourly value for each day of each month and subsequently averaged to provide monthly mean values for each hour. Since Liu and Jordan used a value of 1396 W $\mathrm{m^{-2}}$ and did not average the K_0 values in the same manner, there is an opportunity for slight discrepancies between the results of the two analyses.

The Liu and Jordan type relationship is not particularly well followed at the Canadian locations, judging by the amount of scatter for each of the stations and particularly for Resolute. Moreover, ignoring for the moment the scatter for individual stations, the general relationship is not consistent between



(a) (b) (c) (d)

Fig. 1 Relationship between the ratios of mean hourly values of diffuse to total short-wave radiation at the surface, and of mean hourly values of the total to extra-terrestrial short-wave radiation for (a) Toronto Meteorological Research Station; (b) Montreal Jean Brébeuf; (c) Goose Bay; (d) Resolute

stations. Part of the scatter for individual station data is undoubtedly due to the use of several mean hourly values rather than a single mean daily value for each month, but this is by no means a complete explanation. Resolute (Fig. 1d) is a particularly good example of the nature of the scatter. For any given month a reasonably well-developed relationship exists with the slope being similar from month to month. The overall scatter in the Resolute data is a consequence of a seasonal variation in the constant term (intercept value) in the equation of a straight line representing the relationship for any given month. Highest values occur in the winter months and lowest in the summer months.

This same general pattern appears to hold if one makes a station-to-station comparison, as opposed to a month-to-month comparison for any single station. In this case the tendency is for the intercept value to vary with the latitude of the location. In addition, for any one lower latitude station, the seasonal variation (and hence the scatter) appears to decrease. These two trends can be extended by introducing the relationship derived by Liu and Jordan (1960) for an even lower latitude location, that of Blue Hill (42°13′N). Over the range of ratios being examined here the Blue Hill relationship is approximately linear with a similar slope but an apparent intercept value which is even lower than those of the southern Canadian stations. It is not pertinent to discuss here the relative amount of scatter in the Blue Hill relationship since, as noted earlier there is a difference in the way the data have been averaged. However, the relationship between scatter and latitude implied by the Canadian data alone would tend not to support Liu and Jordan's use of a single line to represent the relationship between the two ratios.

Despite the obvious validity of the Liu and Jordan approach for the Blue Hill data, it is now apparent that such a relationship is inappropriate for Canadian locations. Not only does the relationship undergo spatial variability but also seasonal variability for a given location. The following sections will attempt to determine the cause of this variability and develop a relationship which overcomes the limitations of the Liu and Jordan approach.

3 The influence of multiple reflection

This paper advances the hypothesis that the apparent latitudinal and obvious seasonal variations in the Liu and Jordan relationship are primarily a result of multiple reflection of short-wave radiation between the earth's surface and the overlying atmosphere. The multiply-reflected component has been shown to be an important, though far from dominant, fraction of the total incoming short-wave radiation, especially under conditions of high surface albedo or reflectivity (Hay, 1970; Sawchuck, 1974; Catchpole and Moodie, 1971). The process of multiple reflection is a result of diffuse scattering and it will increase the total incoming short-wave radiation largely as a result of an increase in the diffuse component. Both the ratio expressing the transmission of the atmosphere to the total short-wave radiation and the ratio of diffuse to total short-wave radiation will increase with multiple reflection, though the Liu and Jordan relationship implies that these two ratios are inversely related. The effect of multiple reflection is to displace data points in Fig. 1 further upwards and to the right of the origin compared to their position if multiple reflection were not occurring.

Multiple reflection is a function of both the surface albedo and the atmospheric "back-scatterance" (Möller, 1965). The size of the spatial and temporal variabilities in the former parameter can be seen from the data presented in Table 2. These are "regional" albedos rather than "site" albedos since the

TABLE 2. Monthly mean values of surface (α) and cloud base (α_c) albedos used in study. Source: Hay, 1970

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Toronto	α	50	50	38	28	25	25	25	25	25	25	29	39
Met. Res.	α_c	58	56	53	51	50	48	49	50	53	53	56	57
Toronto	α	50	50	38	28	25	25	25	25	25	25	29	39
Scarborough	α_c	58	56	53	51	50	48	49	50	53	53	56	57
Montreal	α	32	33	25	22	20	20	20	20	20	20	23	28
Jean Brébeuf	α_c	57	55	53	52	50	50	50	50	52	54	57	56
Goose Bay	α	55	56	50	50	35	25	25	25	25	30	35	52
	α_c	56	55	54	56	52	52	53	54	54	57	57	53
Resolute	α	80	80	79	76	69	45	26	21	45	59	74	79
	α_c	57	52	49	45	52	49	51	54	60	60	56	58

multiple reflection process involves surfaces some distance away from the radiation instrument site. Considering the influence of albedo alone, Table 2 indicates that for those stations both the overall relative importance and the intra-annual variability of multiple reflection will be greatest at Resolute.

It appears that variability in the extent of multiple reflection may explain why the Blue Hill relationship is not generally applicable and why the spatial and temporal variabilities in the Liu and Jordan relationship are as described earlier. The following section attempts to reduce these variabilities and increase the specificity of the relationship by at least semi-empirically incorporating the effect of multiple reflection.

4 Modification of the Liu and Jordan approach

A possible modification to the Liu and Jordan approach is to express the relevant ratios in terms of the short-wave radiation fluxes before multiple reflection. For the approach to be useful, these fluxes must themselves be expressed in terms of the observed radiation fluxes which include the effects of multiple reflection. The multiple reflection process is a function of the incoming radiation before multiple reflection occurs ($K{\downarrow}'$), the surface albedo (α) and the atmospheric "back-scatterance" (d) such that:

$$K{\downarrow} = K{\downarrow}' + K{\downarrow}'\alpha d + K{\downarrow}'\alpha^2 d^2 + \ldots . + K{\downarrow}'\alpha^n d^n + \ldots . \qquad (1a)$$

$$= K{\downarrow}'/(1-\alpha d). \qquad (1b)$$

Thus $$K{\downarrow}' = K{\downarrow}\,(1-\alpha d) \qquad (1c)$$

If it is assumed that all the multiply-reflected radiation is diffuse

then $$D{\downarrow} = D{\downarrow}' + K{\downarrow}\alpha d \qquad (2)$$

and $$D{\downarrow}' = D{\downarrow} - K{\downarrow}\alpha d \qquad (3)$$

where $D{\downarrow}'$ is the diffuse short-wave radiation before multiple reflection. The two ratios now become:

$$\frac{K{\downarrow}'}{K_0} = \frac{K{\downarrow}(1-\alpha d)}{K_0} \qquad (4)$$

and

$$\frac{D\downarrow'}{K\downarrow'} = \frac{D\downarrow - K\downarrow\alpha d}{K\downarrow(1 - \alpha d)}\,. \tag{5}$$

The atmospheric back-scatterance (d) may be expressed in terms of the back-scatterance for clear sky (β_0) and the cloud base albedo (α_c), weighted by the proportion of the sky hemisphere which is clear ($1 - n$) and covered by cloud (n), respectively; i.e.,

$$d = n\alpha_c + (1 - n)\beta_0. \tag{6}$$

This modified approach (using the ratios $K\downarrow'/K_0$ and $D\downarrow'/K\downarrow'$ rather than the ratios $K\downarrow/K_0$ and $D\downarrow/K\downarrow$) has been evaluated with the same data as were used in preparation of Fig. 1. Following the results of Möller (1965) β_0 was taken as 0.25. Initially the cloud base albedo was assumed equal to the cloud top albedo enabling the values of the latter parameter as calculated by Hay (1970) to be used in this study. The values used are listed along with the surface "regional" albedo values in Table 2. However, subsequent analyses using a fixed value of cloud base albedo of 0.60 (Davies et al., 1975; London, 1957; Möller, 1965) showed that the calculated values of cloud base albedo did not improve the relationship between the ratios. In fact, in cases where there was some doubt about the validity of the calculated cloud albedos (as in the case of Toronto Scarborough where cloud data for Toronto Malton had to be used), the relationship improved with the use of a constant value of 0.60 for α_c.

It is acknowledged that no attempt has been made to standardize the input data as to the period of record. While the amount of averaging involved will tend to minimize this error it is recognized that the use of unweighted and non-standardized averages will likely account for some of the scatter in the relationships being investigated. This has been accepted as a cost incurred as a result of using published data.

Fig. 2 shows the relationship between $K\downarrow'/K_0$ and $D\downarrow'/K\downarrow'$ for four Canadian stations. As a result of excluding the effect of multiple reflection, both the variability at a single station and the discrepancies between stations have been reduced. Some variability in the relationship does still exist; while part of this may be attributable to the limitations of the data, as noted above, the influence of yet other unaccounted factors cannot be ruled out. Nevertheless consideration of the effect of multiple reflection has obviously resulted in the removal of the largest portion of the variability.

On the basis of the above results, a composite relationship was determined by eye and is presented in Fig. 3. In extrapolating the relationship beyond the limits studied in the previous analysis, the general form of the Liu and Jordan (1960) relationship was maintained since they argued that there is a physical basis for the form of the relationship near its limits; the modifications developed here do not negate these fundamental controls.

The relationship as derived here is based on data from a restricted number of stations and its general spatial applicability requires further independent

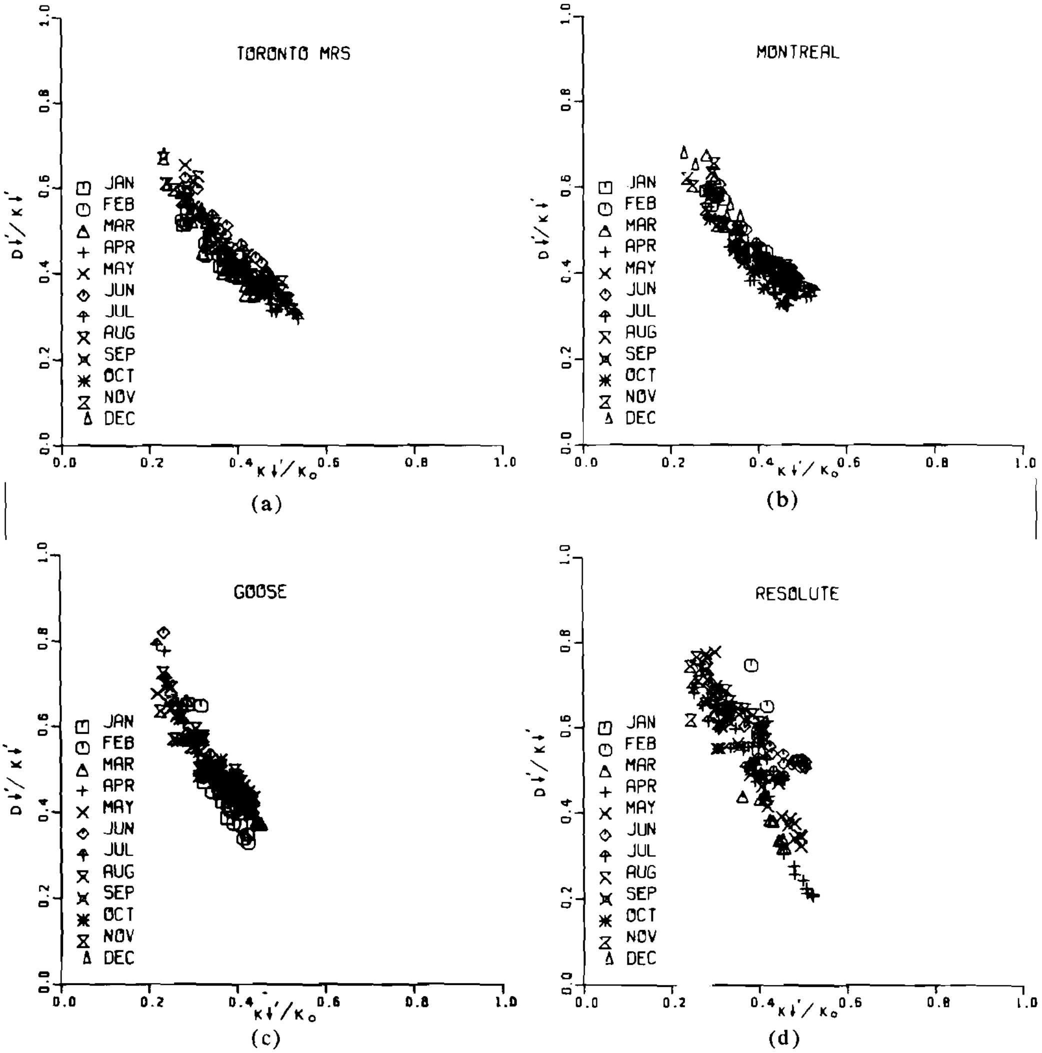


Fig. 2 Relationship between the ratios of mean hourly values of the diffuse to total short-wave radiation at the surface, and of mean hourly values of the total to extra-terrestrial short-wave radiation (excluding the multiply-reflected components of the surface fluxes) for (a) Toronto Meteorological Research Station; (b) Montreal Jean Brébeuf; (c) Goose Bay; (d) Resolute.

verification. The necessary radiation data are not routinely observed at locations other than the four stations already considered and at Toronto Scarborough. Fig. 4 provides a comparison between the observed diffuse radiation at Toronto Scarborough and that calculated using the relationship presented in Fig. 3, a value of 0.60 for α_c, and the relevant "regional" albedos listed in Table 2, and cloud cover data for Toronto International Airport Malton (Atmospheric Environment Service, 1968). The results confirm the validity

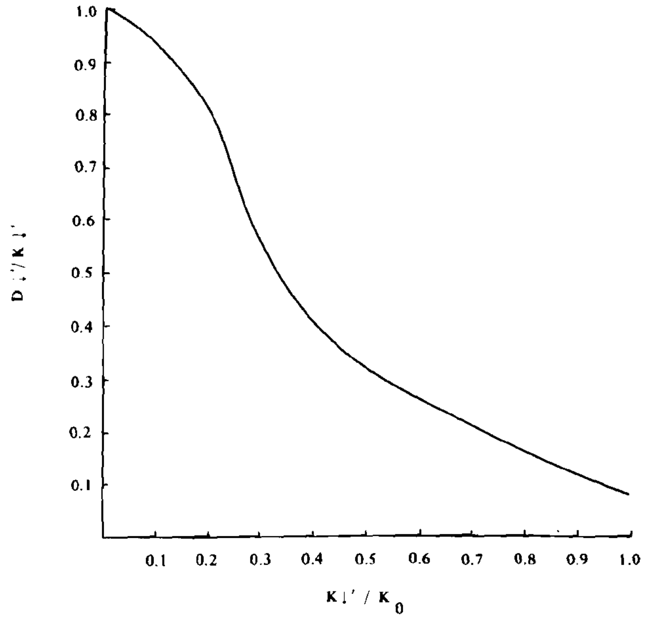


Fig. 3 The general relationship between the ratios of the mean hourly values of the diffuse to total short-wave radiation at the surface, and of mean hourly values of the total to extraterrestrial short-wave radiation (excluding the multiply reflected components of the surface fluxes).

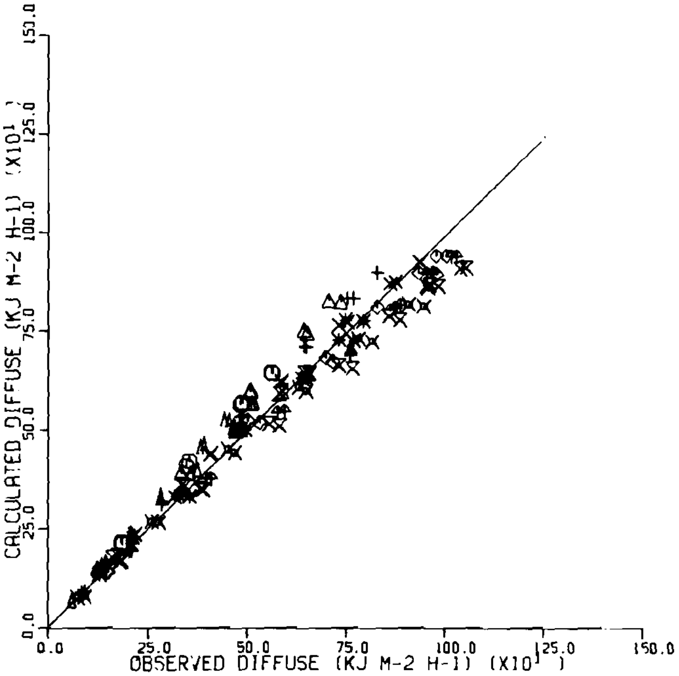


Fig. 4 Comparison of observed and calculated diffuse short-wave radiation for Toronto Scarborough.

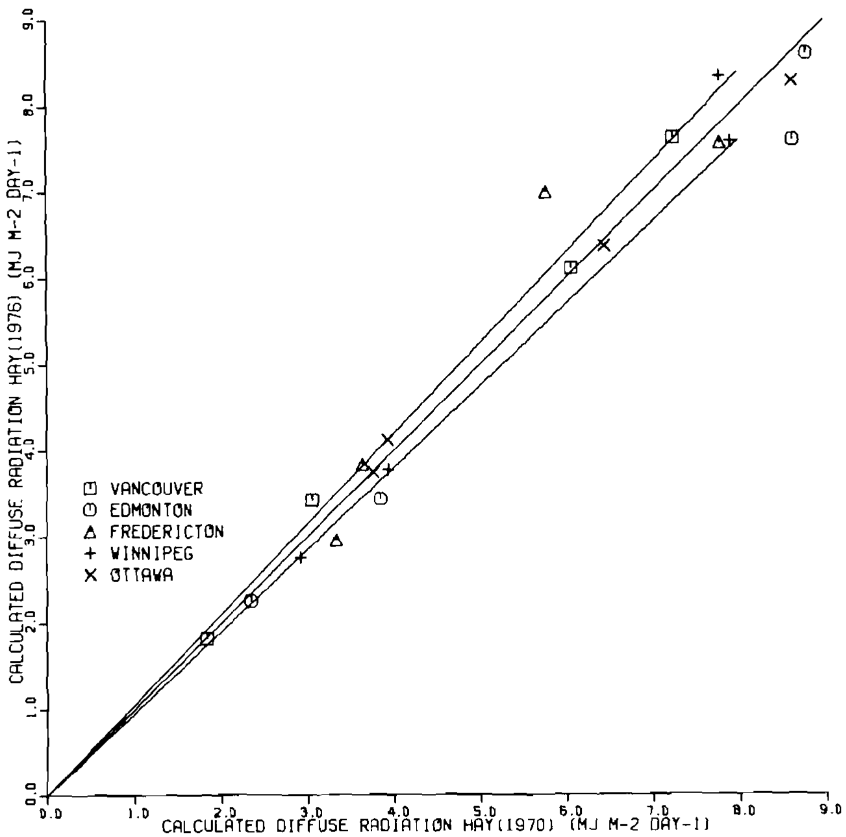


Fig. 5 Comparison of the diffuse short-wave radiation calculated according to Hay (1970) and using the relationship derived in the present study.

of the relationship which allows for multiple reflection, despite the possible inappropriateness of the cloud data.

Hay (1970) has presented maps of the monthly mean total and direct short-wave radiation for Canada from which the relevant data can be obtained for selected locations in order to further test the relationship. The radiation maps are based on calculated rather than observed data but may be considered a totally independent data set with which to test the relationship developed in the present study. Only the "regional" albedos are common to the two approaches since the earlier study calculated the radiation fluxes by the application of appropriate absorption and scattering coefficients to the extraterrestrial radiation.

Fig. 5 presents the results of a comparison of the two independent methods for five locations across southern Canada. The majority of the differences between the two methods are within the indicated limit of 5 per cent, an appropriate accuracy according to Latimer (1972). Thus the widespread regional applicability of the revised relationship is proven.

5 Conclusions

Much of the regional and temporal disparities in the Liu and Jordan type relationship can be attributed to the influence of multiple reflection. With this process taken into account the variability in the relationship is substantially reduced. Its effectiveness in estimating diffuse radiation was evaluated using two independent data sets, with one demonstrating the general regional applicability of the model. The use of constant values for cloud base albedo and the clear sky back-scatterance means that the only additional data requirements are cloud cover and an estimate of "regional" albedo. Given these values, the relationship presented here provides a more general method for calculating the diffuse short-wave radiation from the total short-wave radiation.

Acknowledgments

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